Differentiation and Integration Rules

This handout highlights some of the most frequently encountered rules for differentiation and integration.

For the following let u and v be functions of x, let n be an integer, and let a, c and C be constants.

Fundamental Rules

$$\frac{d(c)}{dx} = 0 \quad\text{———}\quad \int du = u + C$$

$$\frac{d(cu)}{dx} = c\,\frac{du}{dx} \quad\text{———}\quad \int c\,du = c\int du + C$$

$$\frac{d(u+v)}{dx} = \frac{du}{dx} + \frac{dv}{dx} \quad\text{———}\quad \int (du + dv) = \int du + \int dv + C$$

$$\frac{d(u^n)}{dx} = nu^{n-1}\,\frac{du}{dx} \quad\text{———}\quad \int u^n\,du = \frac{u^{n+1}}{n+1} + C$$

$$\frac{d(uv)}{dx} = v\,\frac{du}{dx} + u\,\frac{dv}{dx} \quad\text{———}\quad \int u\,dv = uv - \int v\,du + C$$

$$\frac{d(\frac{u}{v})}{dx} = \frac{v\,\frac{du}{dx} - u\,\frac{dv}{dx}}{v^2}$$

Trigonmetric Functions

$$\frac{d(\sin u)}{dx} = \cos u\,\frac{du}{dx} \quad\text{———}\quad \int \sin u\,du = -\cos u + C$$

$$\frac{d(\cos u)}{dx} = -\sin u\,\frac{du}{dx} \quad\text{———}\quad \int \cos u\,du = \sin u + C$$

$$\frac{d(\tan u)}{dx} = \sec^2 u\,\frac{du}{dx} \quad\text{———}\quad \int \tan u\,du = \ln|\sec u| + C$$

$$\frac{d(\cot u)}{dx} = -\csc^2 u\,\frac{du}{dx} \quad\text{———}\quad \int \cot u\,du = \ln|\sin u| + C$$

$$\frac{d(\sec u)}{dx} = \sec u\,\tan u\,\frac{du}{dx} \quad\text{———}\quad \int \sec u\,du = \ln|\sec u + \tan u| + C$$

Trigonmetric Functions Continued

$$\frac{d(\csc u)}{dx} = -\csc u \cot u \; \frac{du}{dx} \text{ ———— } \int \csc u \; du = -\ln|\csc u \; + \cot u\, | + C$$

$$\int \sec^2 u \; du = \tan u + C \text{ ———— } \int \csc^2 u \; du = -\cot u + C$$

$$\int \sec u \;\; \tan u \; du = \sec u + C \text{ ———— } \int \csc u \;\; \cot u \; du = -\csc u + C$$

Exponential and Logarithmic Functions

$$\frac{d(e^u)}{dx} = e^u \frac{du}{dx} \text{ ———— } \int e^u du = e^u + C$$

$$\frac{d(\ln u)}{dx} = \frac{1}{u}\frac{du}{dx} \text{ ———— } \int \frac{1}{u} du = \ln|u| + C$$

$$\frac{d(a^u)}{dx} = (a^u)(\ln a)\frac{du}{dx} \text{ ———— } \int a^u du = \frac{a^u}{\ln a} + C$$

$$\frac{d(u^v)}{dx} = u^v(v' \ln u + \frac{v}{u} u')$$

Inverse Trigonometric Functions

$$\frac{d(\sin^{-1} u)}{dx} = \frac{1}{\sqrt{1-u^2}} \; \frac{du}{dx} \text{ ———— } \int \frac{1}{\sqrt{a^2-u^2}} = \sin^{-1} \frac{u}{a} + C$$

$$\frac{d(\cos^{-1} u)}{dx} = \frac{-1}{\sqrt{1-u^2}} \; \frac{du}{dx} \text{ ———— } \int \frac{-1}{\sqrt{a^2-u^2}} = -\sin^{-1} \frac{u}{a} + C$$

$$\frac{d(\tan^{-1} u)}{dx} = \frac{1}{1+u^2} \; \frac{du}{dx} \text{ ———— } \int \frac{1}{a^2+u^2} = \frac{1}{a} \tan^{-1} \frac{u}{a} + C$$

$$\frac{d(\cot^{-1} u)}{dx} = \frac{-1}{1+u^2} \; \frac{du}{dx} \text{ ———— } \int \frac{-1}{a^2+u^2} = -\frac{1}{a} \tan^{-1} \frac{u}{a} + C$$

$$\frac{d(\sec^{-1} u)}{dx} = \frac{1}{|u|\sqrt{u^2-1}} \; \frac{du}{dx} \text{ ———— } \int \frac{1}{|u|\sqrt{u^2-a^2}} = \frac{1}{a} \sec^{-1} |\frac{u}{a}| + C$$

$$\frac{d(\csc^{-1} u)}{dx} = \frac{-1}{|u|\sqrt{u^2-1}} \; \frac{du}{dx} \text{ ———— } \int \frac{-1}{|u|\sqrt{u^2-a^2}} = -\frac{1}{a} \sec^{-1} |\frac{u}{a}| + C$$

Hyperbolic Trigonometric Functions

$$\frac{d(\sinh u)}{dx} = \cosh u \, \frac{du}{dx} \text{ ——— } \int \sinh u \; du = \cosh u \; + C$$

$$\frac{d(\cosh u)}{dx} = \sinh u \, \frac{du}{dx} \text{ ——— } \int \cosh u \; du = \sinh u \; + C$$

$$\frac{d(\tanh u)}{dx} = \text{sech}^2 u \, \frac{du}{dx} \text{ ——— } \int \tanh u \; du = -\ln|\text{sech } u \,| \; + C$$

Inverse Hyperbolic Trigonometric Functions

$$\frac{d(\sinh^{-1} u)}{dx} = \frac{1}{\sqrt{1+u^2}} \, \frac{du}{dx} \text{ ——— } \int \frac{1}{\sqrt{1+u^2}} \, du = \sinh^{-1} u + C$$

$$\frac{d(\cosh^{-1} u)}{dx} = \frac{1}{\sqrt{u^2-1}} \, \frac{du}{dx} \text{ ——— } \int \frac{1}{\sqrt{u^2-1}} \, du = \cosh^{-1} u \; + C$$

$$\frac{d(\tanh^{-1} u)}{dx} = \frac{1}{1-u^2} \, \frac{du}{dx} \text{ ——— } \int \frac{1}{1-u^2} \, du = \tanh^{-1} u \; + C$$